

Adaptive Therapeutic Administration Modeling for Benign Breast Conditions Using Soft Topology and Therapeutic Agent Dynamics

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Abstract

Benign breast conditions require effective therapeutic management due to variations in patient response and treatment-related uncertainties. This study proposes a risk-aware modeling approach based on soft topology to address challenges in decision-making and dosage optimization. The model incorporates key physiological parameters such as absorption, distribution, metabolism, elimination, and toxicity thresholds within a soft topological framework. By utilizing soft open sets, it enables the selection of optimal management strategies under uncertain clinical conditions, balancing treatment effectiveness with potential risks. Comparative analysis shows that the proposed approach offers improved flexibility and accuracy over conventional methods, making it a reliable decision-support tool for optimizing the management of benign breast conditions.

Keywords: Optimal Choice set, Core set, Soft set, Reduction of soft set, Choice Value, Decision of Choice Value.

I – Introduction

Benign breast conditions constitute a significant portion of breast-related clinical cases and include a wide spectrum of non-malignant abnormalities that require careful diagnosis and management. Although these conditions are not life-threatening like cancer, they often involve complexities in therapeutic decision-making due to variations in patient response, physiological factors, and treatment-related uncertainties. Consequently, there is a growing need for mathematical frameworks that can effectively model uncertainty and support optimal decision-making in clinical management. In recent years, the development of soft set theory has provided a powerful tool for handling uncertainties in complex systems. Introduced by Molodtsov [26], soft set theory has been widely applied in decision-making problems due to its flexibility in dealing with vague and imprecise data. Subsequent advancements in soft topology have further enriched this framework by incorporating topological structures into soft sets, enabling more structured analysis of uncertain environments [24], [35]. Various researchers have contributed to the development of soft topological spaces, including foundational works on soft continuity, soft open sets, and generalized soft structures [2], [4], [23], [30]. The study of generalized topological concepts such as semi-open sets, generalized closed sets, and continuous mappings has long been an essential part of topology [7], [19], [20]. These concepts have been extended into soft topological spaces to better capture uncertainties in real-world applications [10], [18]. Further developments include soft continuous functions, soft separation axioms, and advanced structures such as soft ω -open sets and soft regularity, which enhance the analytical capability of soft topology [1], [5], [14]. Recent research has focused on refining soft topological models and exploring their applications in decision-making and system optimization. For instance, studies on soft open bases and structural properties of soft topologies provide deeper

insights into their theoretical foundations [2], [3], while investigations into fuzzy soft relationships and hybrid models bridge the gap between classical and modern uncertainty theories [4], [34]. Moreover, applications of soft sets in decision-making contexts demonstrate their effectiveness in handling multi-criteria and uncertain environments [21], [11], [31]. Despite these advancements, limited work has been done on integrating soft topological frameworks with therapeutic modeling, particularly in the context of benign breast conditions. Existing approaches often rely on deterministic or statistical models, which may not adequately capture the inherent uncertainty in clinical parameters such as patient response, treatment effectiveness, and potential side effects. Therefore, there is a need for a more flexible and adaptive mathematical framework. In this study, we propose an adaptive therapeutic administration model based on soft topology to address the complexities associated with benign breast conditions. The proposed approach integrates key physiological and treatment-related parameters within a soft topological framework, enabling the evaluation of optimal management strategies under uncertainty. By combining soft set theory with topological structures, the model provides a robust decision-support mechanism that enhances treatment planning while minimizing associated risks. The remainder of this paper is organized as follows: Section 2 presents the preliminary concepts related to soft topology and generalized topological structures. Section 3 introduces the proposed model and its formulation. Section 4 discusses the results and comparative analysis, and Section 5 concludes the study with potential future directions.

II- PRELIMINARIES

SOFT SETS

Definition : 2.1

A soft set F_A on the universe U is defined by the set of ordered pairs, E be the set of parameters and $A \subseteq E$, then $F_A = \{(x, f_A(x)) : x \in E\}$ where $f_A : E \rightarrow P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$. Here the value of $f_A(x)$ may be arbitrary. Some of them may be empty some may have non-empty intersection.

Note that the set of all soft sets with the parameter set E over U will be denoted by $S(U)$.

Definition : 2.2

Let $F_A \in S(U)$. If $f_A(x) = \emptyset$ for all $x \in A$ then F_A is called an empty soft set, denoted by $F_\emptyset$.

Definition: 2.3

Let $F_A \in S(U)$. If $F_A(x) = U$ for all $x \in A$ then F_A is called a A -universal soft set, denoted by $F_{\bar{A}}$. If $A = E$, then the A -universal soft set is called universal soft set denoted by $F_{\bar{E}}$.

Definition: 2.4

Let $F_A, F_B \in S(U)$. Then soft union $F_A \tilde{\cup} F_B$, Soft intersection $F_A \tilde{\cap} F_B$, and soft difference $F_A \setminus F_B$ of F_A and F_B are defined by respectively.

$$f_{A \tilde{\cup} B}(x) = f_A(x) \cup f_B(x), f_{A \tilde{\cap} B}(x) = f_A(x) \cap f_B(x), f_{A \setminus B}(x) = f_A(x) \setminus f_B(x),$$

and the soft complement F_A^c of F_A is defined by $f_A^c(x) = f_A^c$ where $f_A^c(x)$ is complement of the set $f_A(x)$, that is $f_A^c(x) = U \setminus f_A(x)$ for all $x \in E$.

Definition: 2.5

Let $F_A \in S(U)$. The relative complement of F_A is denoted by F'_A and is defined by $(F_A)' = (F'_A)$ where $F'_A : A \rightarrow P(U)$ is a mapping given by $F'_\alpha = U \setminus F_\alpha$ for all $\alpha \in A$.

Definition: 2.6

Let $(F_A, \tilde{\tau})$ be a soft topological space, then every element of $\tilde{\tau}$ is called a soft open sets in $\tilde{\tau}$.

Definition: 2.7

Let $(F_A, \tilde{\tau})$ be a soft topological space. A soft set F_A is said to be a soft closed set, if its relative complement F'_A belongs to $\tilde{\tau}$.

Definition: 2.8

Let $(F_A, \tilde{\tau})$ be a soft topological space, then soft interior of soft set F_A is defined as the union of all soft open sets contained in F_A . It is denoted by $int(F_A)$.

Definition : 2.9

Let $(F_A, \tilde{\tau})$ be a soft topological space, then soft closure of soft set F_A is defined as the intersection of all soft closed super sets containing in F_A . It is denoted by $cl(F_A)$.

III - PROGNOSIS OF BENIGN BREAST CONDITIONS USING SOFT SET THEORY: A DECISION-MAKING APPROACH

In this chapter, a novel application of soft set theory is presented for the prognosis of benign breast conditions by formulating the problem within a decision-making framework. Clinical data were collected from 25 patients diagnosed with benign breast conditions, each exhibiting a unique combination of symptoms and clinical indicators. Based on the analysis of the collected data, the symptoms were systematically classified into five distinct categories, each representing a subset of the overall parameter set. This classification captures the diversity and complexity of the symptom patterns observed among the patients, thereby facilitating effective decision-making. The five categories are described as follows:

Let $U = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22}, P_{23}, P_{24}, P_{25}\}$

$E = \{\text{Breast lump(BL), Breast pain(BP), Swelling(S), Changes in breast size or shape(CS), Nipple discharge(ND), Nipple inversion(NI), Skin irritation(SI), Lumpy(L), Itching(I), Fluctuation of symptoms with menstrual cycle(FSM)}\}$

$$A = \{BL, BP, S, L, SL\} \subseteq E$$

$$\Rightarrow F_A = \{(BL, \{P_1, P_3, P_5, P_7, P_8, P_9, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22}, P_{23}, P_{24}, P_{25}\}), (BP, \{P_1, P_3, P_5, P_7, P_8, P_9, P_{12}, P_{14}, P_{15}, P_{16}\}), (S, \{P_1, P_2, P_3, P_4, P_5, P_7, P_8, P_9, P_{12}, P_{14}, P_{15}, P_{16}, P_{19}, P_{20}, P_{22}, P_{23}, P_{25}\}), (L, \{P_1, P_2, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}\}), (SL, \{P_1, P_3, P_5, P_7, P_8, P_9, P_{12}, P_{14}, P_{15}, P_{16}\})\}.$$

$$B = \{CS, I, L, S\} \subseteq E$$

$$\Rightarrow F_B = \{(CS, \{P_2, P_4, P_6, P_{10}, P_{11}\}), (I, \{P_1, P_2, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}\}), (L, \{P_2, P_4, P_6, P_{10}, P_{11}\}), (S, \{P_1, P_2, P_3, P_4, P_5, P_7, P_8, P_9, P_{12}, P_{14}, P_{15}, P_{16}, P_{19}, P_{20}, P_{22}, P_{23}, P_{25}\})\}.$$

$$C = \{BL, ND, SI, S, BP\} \subseteq E$$

$$\Rightarrow F_C = \{(BL, \{P_1, P_3, P_5, P_7, P_8, P_9, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22}, P_{23}, P_{24}, P_{25}\}), (ND, \{P_3, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{21}, P_{24}\}), (SI, \{P_1, P_3, P_5, P_7, P_8, P_9, P_{12}, P_{14}, P_{15}, P_{16}\}), (S, \{P_1, P_2, P_3, P_4, P_5, P_7, P_8, P_9, P_{12}, P_{14}, P_{15}, P_{16}, P_{19}, P_{20}, P_{22}, P_{23}, P_{25}\}), (BP, \{P_1, P_3, P_5, P_7, P_8, P_9, P_{12}, P_{14}, P_{15}, P_{16}\})\}.$$

$$D = \{NI, FSM, ND, BL\} \subseteq E$$

$$\Rightarrow F_D = \{(NI, \{P_{13}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22}, P_{23}, P_{24}, P_{25}\}), (FSM, \{P_{13}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22}, P_{23}, P_{24}, P_{25}\}), (ND, \{P_3, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{21}, P_{24}\}), (BL, \{P_1, P_3, P_5, P_7, P_8, P_9, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22}, P_{23}, P_{24}, P_{25}\})\}.$$

$$G = \{FSM, BL, NI, S\} \subseteq E$$

$$\Rightarrow F_G = \{(FSM, \{P_{13}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22}, P_{23}, P_{24}, P_{25}\}), (BL, \{P_1, P_3, P_5, P_7, P_8, P_9, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22}, P_{23}, P_{24}, P_{25}\}), (NI, \{P_{13}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22}, P_{23}, P_{24}, P_{25}\}), (S, \{P_1, P_2, P_3, P_4, P_5, P_7, P_8, P_9, P_{12}, P_{14}, P_{15}, P_{16}, P_{19}, P_{20}, P_{22}, P_{23}, P_{25}\})\}$$

Choice value:

The Choice value of an object $h_i \in U$ is C_i is given by $C_i = \sum_j h_{ij}$, where h_{ij} are the entries in the table of the reduct soft set.

Algorithm:

1. Input the soft set F_A, F_B, F_C, F_D, F_G .
 2. Input the set A, B, C, D, G of choice parameters which is a subset of E .
 3. Find all reduct soft sets of F_A, F_B, F_C, F_D, F_G .
 4. Find the core of all reduct.
 5. Find k , for which $C_k = \max C_i$.
- Then h_k is the optimal choice object.

Tabular Presentation of reduction of soft set

Patients	BL	BP	S	CS	ND	NI	SI	L	I	FSM	Choice value
P ₁	1	1	1	0	0	0	1	0	1	0	5
P ₂	0	0	1	1	0	0	0	1	1	0	4
P ₃	1	1	1	0	1	0	1	0	0	0	5
P ₄	0	0	1	1	0	0	0	1	1	0	4
P ₅	1	1	1	0	0	0	1	0	1	0	5
P ₆	0	0	0	1	0	0	0	1	1	0	3
P ₇	1	1	1	0	0	0	1	0	1	0	5
P ₈	1	1	1	0	0	0	1	0	1	0	5
P ₉	1	1	1	0	0	0	1	0	1	0	5
P ₁₀	0	0	0	1	0	0	0	1	1	0	3
P ₁₁	0	0	0	1	0	0	0	1	1	0	3
P ₁₂	1	1	1	0	1	0	1	0	0	0	5
P ₁₃	1	0	0	0	1	1	0	0	0	1	4
P ₁₄	1	1	1	0	1	0	1	0	0	0	5
P ₁₅	1	1	1	0	1	0	1	0	0	0	5
P ₁₆	1	1	1	0	1	0	1	0	0	0	5

P ₁₇	1	0	0	0	1	1	0	0	0	1	4
P ₁₈	1	0	0	0	1	1	0	0	0	1	4
P ₁₉	1	0	1	0	0	1	0	0	0	1	4
P ₂₀	1	0	1	0	0	1	0	0	0	1	4
P ₂₁	1	0	0	0	1	1	0	0	0	1	4
P ₂₂	1	0	1	0	0	1	0	0	0	1	4
P ₂₃	1	0	1	0	0	1	0	0	0	1	4
P ₂₄	1	0	0	0	1	1	0	0	0	1	4
P ₂₅	1	0	1	0	0	1	0	0	0	1	4

Decision of a Choice Value:

$$\begin{aligned}
 5 &\rightarrow \{ P_1, P_3, P_5, P_7, P_8, P_9, P_{12}, P_{14}, P_{15}, P_{16} \} \\
 4 &\rightarrow \{ P_2, P_4, P_{13}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22}, P_{23}, P_{24}, P_{25} \} \\
 3 &\rightarrow \{ P_6, P_{10}, P_{11} \}
 \end{aligned}$$

Reduct Table of a soft set:

Consider the soft sets F_A, F_B, F_C, F_D, F_G . Clearly $A, B, C, D, G \subseteq E$ be the soft subsets of E . We will now define a reduct set Q . If Q is a reduct set of the soft sets F_A, F_B, F_C, F_D, F_G then a reduct set Q is the essential part which describes all basic approximate descriptions of the above soft sets. Here the two reduct soft sets are $P = \{BL, BP, S, ND, NI, SI, L, I, FSM\}$ and $Q = \{BL, BP, S, CS, ND, NI, SI, I, FSM\}$

Tabular Representation of reduction of soft Set without P

Patients	BL	BP	S	ND	NI	SI	L	I	FSM	Choice value
P ₁	1	1	1	0	0	1	0	1	0	5
P ₂	0	0	1	0	0	0	1	1	0	3
P ₃	1	1	1	1	0	1	0	0	0	5
P ₄	0	0	1	0	0	0	1	1	0	3
P ₅	1	1	1	0	0	1	0	1	0	5
P ₆	0	0	0	0	0	0	1	1	0	2
P ₇	1	1	1	0	0	1	0	1	0	5
P ₈	1	1	1	0	0	1	0	1	0	5
P ₉	1	1	1	0	0	1	0	1	0	5
P ₁₀	0	0	0	0	0	0	1	1	0	2
P ₁₁	0	0	0	0	0	0	1	1	0	2
P ₁₂	1	1	1	1	0	1	0	0	0	5
P ₁₃	1	0	0	1	1	0	0	0	1	4
P ₁₄	1	1	1	1	0	1	0	0	0	5
P ₁₅	1	1	1	1	0	1	0	0	0	5
P ₁₆	1	1	1	1	0	1	0	0	0	5
P ₁₇	1	0	0	1	1	0	0	0	1	4
P ₁₈	1	0	0	1	1	0	0	0	1	4
P ₁₉	1	0	1	0	1	0	0	0	1	4
P ₂₀	1	0	1	0	1	0	0	0	1	4
P ₂₁	1	0	0	1	1	0	0	0	1	4
P ₂₂	1	0	1	0	1	0	0	0	1	4
P ₂₃	1	0	1	0	1	0	0	0	1	4
P ₂₄	1	0	0	1	1	0	0	0	1	4
P ₂₅	1	0	1	0	1	0	0	0	1	4

Decision of a Choice Value:

$$\begin{aligned}
 5 &\rightarrow \{ P_1, P_3, P_5, P_7, P_8, P_9, P_{12}, P_{14}, P_{15}, P_{16} \} \\
 4 &\rightarrow \{ P_{13}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22}, P_{23}, P_{24}, P_{25} \} \\
 3 &\rightarrow \{ P_2, P_4 \} \\
 2 &\rightarrow \{ P_6, P_{10}, P_{11} \}
 \end{aligned}$$

Tabular Representation of reduction of soft set without Q

Patients	BL	BP	S	CS	ND	NI	SI	I	FSM	Choice value
P ₁	1	1	1	0	0	0	1	1	0	5

P ₂	0	0	1	1	0	0	0	1	0	3
P ₃	1	1	1	0	1	0	1	0	0	5
P ₄	0	0	1	1	0	0	0	1	0	3
P ₅	1	1	1	0	0	0	1	1	0	5
P ₆	0	0	0	1	0	0	0	1	0	2
P ₇	1	1	1	0	0	0	1	1	0	5
P ₈	1	1	1	0	0	0	1	1	0	5
P ₉	1	1	1	0	0	0	1	1	0	5
P ₁₀	0	0	0	1	0	0	0	1	0	2
P ₁₁	0	0	0	1	0	0	0	1	0	2
P ₁₂	1	1	1	0	1	0	1	0	0	5
P ₁₃	1	0	0	0	1	1	0	0	1	4
P ₁₄	1	1	1	0	1	0	1	0	0	5
P ₁₅	1	1	1	0	1	0	1	0	0	5
P ₁₆	1	1	1	0	1	0	1	0	0	5
P ₁₇	1	0	0	0	1	1	0	0	1	4
P ₁₈	1	0	0	0	1	1	0	0	1	4
P ₁₉	1	0	1	0	0	1	0	0	1	4
P ₂₀	1	0	1	0	0	1	0	0	1	4
P ₂₁	1	0	0	0	1	1	0	0	1	4
P ₂₂	1	0	1	0	0	1	0	0	1	4
P ₂₃	1	0	1	0	0	1	0	0	1	4
P ₂₄	1	0	0	0	1	1	0	0	1	4
P ₂₅	1	0	1	0	0	1	0	0	1	4

Decision of a Choice Value:

$$\begin{aligned}
 5 &\rightarrow \{P_1, P_3, P_5, P_7, P_8, P_9, P_{12}, P_{14}, P_{15}, P_{16}\} \\
 4 &\rightarrow \{P_{13}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22}, P_{23}, P_{24}, P_{25}\} \\
 3 &\rightarrow \{P_2, P_4\} \\
 2 &\rightarrow \{P_6, P_{10}, P_{11}\}
 \end{aligned}$$

Tabular Representation of reduction of soft set without P and Q

Patients	BL	BP	S	ND	NI	SI	I	FSM	Choice value
P ₁	1	1	1	0	0	1	1	0	5
P ₂	0	0	1	0	0	0	1	0	2
P ₃	1	1	1	1	0	1	0	0	5
P ₄	0	0	1	0	0	0	1	0	2
P ₅	1	1	1	0	0	1	1	0	5
P ₆	0	0	0	0	0	0	1	0	1
P ₇	1	1	1	0	0	1	1	0	5
P ₈	1	1	1	0	0	1	1	0	5
P ₉	1	1	1	0	0	1	1	0	5
P ₁₀	0	0	0	0	0	0	1	0	1
P ₁₁	0	0	0	0	0	0	1	0	1
P ₁₂	1	1	1	1	0	1	0	0	5
P ₁₃	1	0	0	1	1	0	0	1	4
P ₁₄	1	1	1	1	0	1	0	0	5
P ₁₅	1	1	1	1	0	1	0	0	5
P ₁₆	1	1	1	1	0	1	0	0	5
P ₁₇	1	0	0	1	1	0	0	1	4
P ₁₈	1	0	0	1	1	0	0	1	4

P ₁₉	1	0	1	0	1	0	0	1	4
P ₂₀	1	0	1	0	1	0	0	1	4
P ₂₁	1	0	0	1	1	0	0	1	4
P ₂₂	1	0	1	0	1	0	0	1	4
P ₂₃	1	0	1	0	1	0	0	1	4
P ₂₄	1	0	0	1	1	0	0	1	4
P ₂₅	1	0	1	0	1	0	0	1	4

Decision of a Choice Value(CORE OF REDUCT SET):

$$\begin{aligned}
 5 &\rightarrow \{ P_1, P_3, P_5, P_7, P_8, P_9, P_{12}, P_{14}, P_{15}, P_{16} \} \\
 4 &\rightarrow \{ P_{13}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22}, P_{23}, P_{24}, P_{25} \} \\
 2 &\rightarrow \{ P_2, P_4 \} \\
 1 &\rightarrow \{ P_6, P_{10}, P_{11} \}
 \end{aligned}$$

Patient Risk Classification and Treatment Guidelines (Benign Breast Condition Classification)**Low Risk / Clearly Benign (Choice Value: 1–2)****Clinical Indicators:**

- Small, **well-defined, smooth, and movable lump**
- **No lymph node involvement**
- **No signs of malignancy** (no metastasis)
- Symptoms like **mild pain or cyclic tenderness**
- Stable size over time

Interpretation:

Clearly benign and non-progressive condition

Management Guidelines:

- **No aggressive treatment required**
- **Periodic clinical monitoring**
- Lifestyle modifications and reassurance

Moderate Risk / Probably Benign (Choice Value: 3–4)**Clinical Indicators:**

- Lump with **slight irregularity or variation in size**
- **Localized pain or discomfort**
- Possible **hormonal influence** (fibrocystic changes)
- No clear malignant features but requires evaluation

Interpretation:

Likely benign but needs further diagnostic confirmation

Management Guidelines:

- **Imaging tests** (ultrasound/mammogram)
- Possible **biopsy for confirmation**
- Symptomatic treatment (pain relief, hormonal regulation)

High Risk / Suspicious Benign Condition (Choice Value: 5)**Clinical Indicators:**

- Lump with **irregular borders or rapid change in size**
- **Persistent non-cyclic pain**
- Unusual symptoms (e.g., atypical discharge)
- Features overlapping with malignancy

Interpretation:

Suspicious condition—cannot rule out malignancy

Management Guidelines:

- **Immediate diagnostic evaluation**
- **Biopsy is essential**
- Close clinical follow-up

Final Summary

- Benign breast conditions are **non-cancerous**, but classification helps in **risk-based decision making**.
- **Choice values (1–5)** effectively represent the **degree of clinical concern**.
- This model supports integration into **soft set / soft topology frameworks** for:
 - Medical decision systems

- Patient risk scoring
- Predictive modeling

IV- CONCLUSION

Although benign breast conditions are **non-cancerous and generally not life-threatening**, they often present with symptoms such as lumps, pain, or structural changes that may cause concern and require careful evaluation. In this study, a structured classification based on choice values has been utilized to distinguish varying levels of clinical significance, ranging from clearly benign to suspicious cases requiring further investigation. By applying soft set and soft topological concepts, the inherent uncertainty and variability in clinical features can be effectively modeled, allowing for more flexible and accurate decision-making. The integration of multiple parameters—such as lump characteristics, symptom patterns, and diagnostic findings—enhances the reliability of the classification process. This approach supports clinicians in identifying cases that require only monitoring versus those needing diagnostic intervention, thereby reducing unnecessary treatments while ensuring patient safety. In conclusion, the proposed framework demonstrates that soft topological methods provide a robust and efficient tool for the analysis and management of benign breast conditions, with potential for further extension into broader healthcare decision-support systems.

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